Differential Formulation of On-Surface Measured Equation of Invariance for 2-D Conducting Scatterings

Y. W. Liu, K. K. Mei, and K. N. Yung

***Abstract*—In this letter formulations of on-surface measured equation of invariance (OSMEI) method for two-dimensional (2-D) conducting scatterings are derived from scalar wave equations rather than integral equations. At the same time, the MEI equations for TM and TE cases are proved to be same. As long as one of the MEI equations of the above two cases is found, the MEI equation of another case is automatically given. A great advantage of the OSMEI method over the conventional boundary integration or differential equation method is that the OSMEI can solve conducting scattering problems efficiently. Examples of conducting circular and rectangular cylinder scatterings for both TM and TE case show that results of the OSMEI are in excellent agreement with those of the method of moments (MoM).**

I. INTRODUCTION

IN THE AREA of electromagnetic computations, a number of fast computation methods have been proposed in recent years. In the context of integral equations (IE), most of the effort is focused on generating sparse rather than dense matrices to accelerate the speed of solving the matrices. Most notably, there are wavelet expansion [1], fast multipole method [2], and impedance matrix localization [3], etc. Whereas, in the context of differential equations (DE's), most of the effort is concentrated on moving the truncated boundary as close as possible to the scatterer to reduce the number of unknowns. The recently developed method of measured equation of invariance (MEI) [4] has succeeded in the latter quite well. Originally, the objective of MEI was to terminate the finite-difference (FD) or finite-element (FE) mesh close to the scatterer surface, but it is now merged into the area of integral equations to generate the sparse matrix by using the reciprocity theorem [5]. In [5], the integral equation generated MEI is on surface, i.e., there is no FD or FE equation required, therefore the number of unknowns is identical to that of the method of moments (MoM). In this letter, we directly discretize scalar wave equations to obtain formulations of on-surface measured equation of invariance (OSMEI). Furthermore, we prove that both TM and TE cases have the same MEI formulation. The MEI equation of the TM case can be used in the TE case, and vice verse. A great advantage of the OSMEI over the conventional IE and DE methods is that the OSMEI generates the least number of the unknowns and a cyclic band sparse matrix. Since fewer number of the unknowns saves computational memory and the sparse matrix accelerates computational speed, the OSMEI is a truly fast computational method. In order to demonstrate the validity of the OSMEI method, scatterings of conducting circular and rectangular cylinders for both TM and TE plane wave incidence cases are chosen as examples in comparing with the results of MoM. Excellent agreement between the results of the OSMEI and MoM is reached.

Manuscript received October 7, 1997. This work was supported in part by the City University of Hong Kong Strategic Research Grant 7000595-570 and by Hong Kong Research Grant Council Grant CERG 9040237.

The authors are with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong.

Publisher Item Identifier S 1051-8207(98)01466-4.

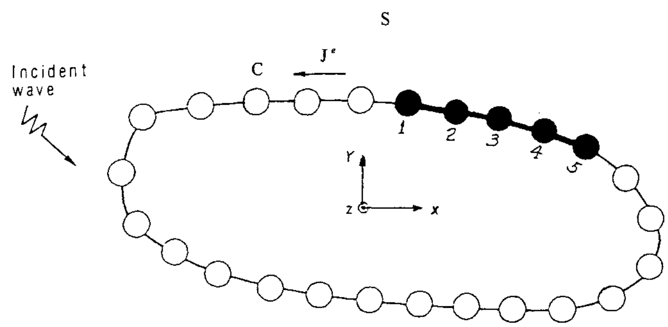

Fig. 1. Surface mesh of a perfect conducting scatterer for the OSMEI.

II. OSMEI METHOD

For an easy understanding, let us consider a perfectly conducting cylindrical scatterer, as shown in Fig. 1. This cylinder is uniform along the z axes and has an arbitrary cross section described by a close contour C on the x-y plane. The exterior space region of the scatterer is denoted by S. Then, the problem is to calculate the scattering field generated by a plane wave illuminating on the scatterer at an arbitrary angle. We further assume that such an incident plane wave can be expressed by the scalar wave function E_z or H_z. We start to discretize the following scalar wave equations of scattered fields:

$$\left(\frac{\partial^2}{\partial n^2} + \frac{\partial^2}{\partial l^2} + \kappa^2\right)\begin{Bmatrix} E_z^s \\ H_z^s \end{Bmatrix} = 0 \tag{1}$$

where n is normal to the surface and l is tangential to the surface. E_z is for the TM case and H_z for the TE case. The second-order tangential partial differential in (1) can be

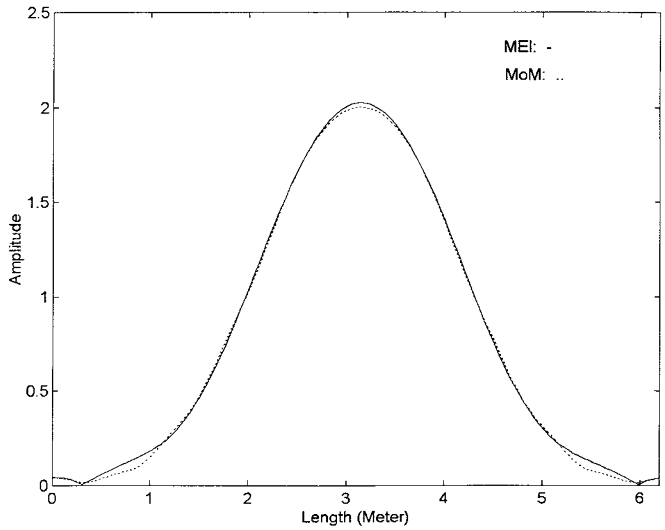

Fig. 2. TM surface currents of the circular cylinder.

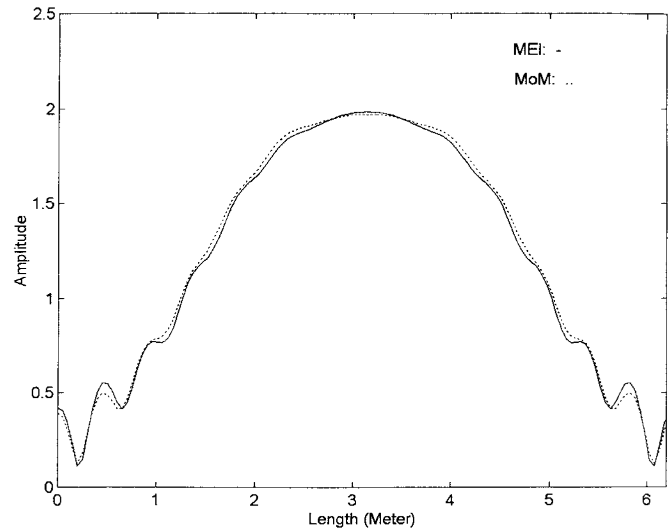

Fig. 3. TE surface currents of the circular cylinder.

discretized directly as

$$\left(\frac{\partial^2}{\partial l^2}+\kappa^2\right)\begin{Bmatrix}E_z^s\\ H_z^s\end{Bmatrix}\approx\sum_{i=1}^{N}\begin{Bmatrix}b_i^e E_{zi}^s\\ a_i^m H_{zi}^s\end{Bmatrix},\qquad N\geq 3. \tag{2}$$

The second-order normal partial differential in (1) can be written as

$$\frac{\partial^2}{\partial n^2}\begin{Bmatrix}E_z^s\\ H_z^s\end{Bmatrix}=\frac{\partial}{\partial n}\begin{Bmatrix}\partial E_z^s/\partial n\\ \partial H_z^s/\partial n\end{Bmatrix}$$
$$=\begin{Bmatrix}-j/(k\eta)\\ j\eta/k\end{Bmatrix}\frac{\partial}{\partial n}\begin{Bmatrix}H_l^s\\ E_l^s\end{Bmatrix}\approx\sum_{i=1}^{N}\begin{Bmatrix}a_i^e H_{li}^s\\ b_i^m E_{li}^s\end{Bmatrix}. \tag{3}$$

In (3), we approximate $\partial/\partial n$ of H_l^s (E_l^s) by using the linear combination of the H_l^s (E_l^s) and nearby fields [6]. Substituting (2) and (3) into (1), we get on-surface measured equation of invariance as follows:

$$\sum_{i=1}^{N}\begin{Bmatrix}a_i^e H_{li}^s\\ a_i^m H_{zi}^s\end{Bmatrix}+\sum_{i=1}^{N}\begin{Bmatrix}b_i^e E_{zi}^s\\ b_i^m E_{li}^s\end{Bmatrix}=0. \tag{4}$$

Equation (4) is called the MEI equations. The TM coefficients $\{a_i^e\},\{b_i^e\}$ and TE $\{a_i^m\},\{b_i^m\}$ $(i=1,2,\cdots,N)$ in (4) are called as MEI coefficients. Only $(2N-1)$ coefficients are independent. By means of at least $(2N-1)$ possible solutions of E_z^s, H_l^s, H_z^s, and E_l^s, the coefficients $\{a_i^e\},\{b_i^e\},\{a_i^m\}$, and $\{b_i^m\}$ can be numerically found.

For the TM case [7]

$$E_z^s(\boldsymbol{r})=-\frac{k\eta}{4}\int_c J_z(r')H_0^{(2)}(k|r-r'|)\,d\ell' \tag{5}$$

$$H_\ell^s(\boldsymbol{r})=-j\frac{\omega\mu}{k^2}\frac{\partial E_z^s}{\partial n}=-\frac{1}{4j}\int_c J_z(r')\frac{\partial H_0^{(2)}(k|r-r'|)}{\partial n}\,d\ell' \tag{6}$$

and for the TE case [7]

$$H_z^s(\boldsymbol{r})=\frac{1}{4j}\int_c(\overline{i_z}\cdot\nabla'\times\overline{J(r')})H_0^{(2)}(k|r-r'|)\,d\ell' \tag{7}$$
$$=\frac{1}{4j}\int_c\frac{\partial J_l(r')}{\partial n'}H_0^{(2)}(k|r-r'|)\,d\ell'$$
$$=\frac{1}{4j}\int_c J_{\text{eq}}(r')H_0^{(2)}(k|r-r'|)\,d\ell'$$

$$E_\ell^s(\boldsymbol{r})=\frac{\omega\mu}{k^2}\frac{\partial H_z^s}{\partial n}=\frac{\eta}{4k}\int_c J_{\text{eq}}(r')\frac{\partial H_0^{(2)}(k|r-r'|)}{\partial n}\,d\ell' \tag{8}$$

where k is the wavenumber, ω is angle frequency, μ is permeability of free space, and η is the intrinsic impedance of the free space. $J_z(r')$ is the TM electric current and $J_{\text{eq}}(r')=\partial J_l(r')/\partial n'$ is the TE equivalent current. The J_z and J_{eq} are known as "metrons" and are usually taken as

$$J(\ell')=\exp(j2\pi n\ell'/L),\qquad n=0,\pm1,\pm2,\cdots \tag{9}$$

where L is the perimeter of the scatterer surface C, and ℓ' is the source position on the C.

Substituting (5)–(8) into (4), we easily prove that the MEI's equation for TM case is the same as the MEI's equation for TE case, i.e., $\{a_i^e\}=\{a_i^m\}$, and $\{b_i^e\}=\{b_i^m\}$. That is to say, we need to find only one set of the MEI coefficients $\{a_i\}$ and $\{b_i\}$ (either TM or TE). Using (4) and boundary conditions of the conducting surface at each node, we finally generate the following matrix equations:

$$[A]\underline{H_\ell^s}=[B]\underline{E_z^{\text{in}}},\qquad \text{for TM} \tag{10}$$
$$[A]\underline{H_z^s}=[B](j\eta/k)\partial\underline{H_z^{\text{in}}}/\partial n,\qquad \text{for TE} \tag{11}$$

where $[A]$ and $[B]$ are the cyclic band matrices with bandwidth N. Vector $\underline{E_z^{\text{in}}}$ and $\underline{H_z^{\text{in}}}$ represent real incident wave on the scatterer surface. Vector $\underline{H_\ell^s}$ and $\underline{H_z^s}$ stand for unknown tangential magnetic field components.

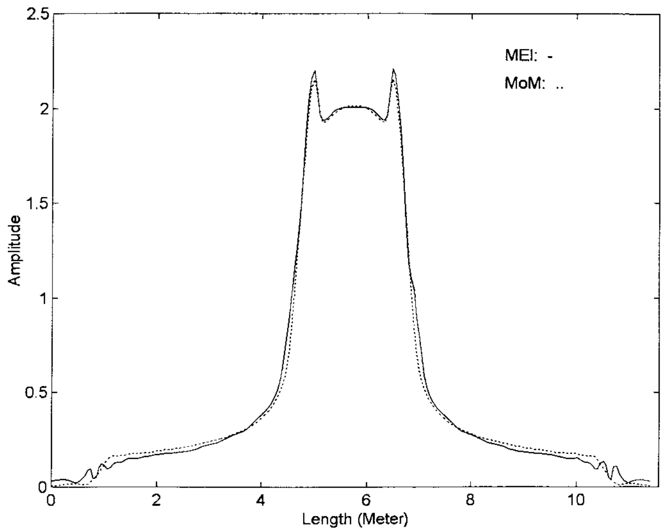

Fig. 4. TM surface currents of the rectangular cylinder.

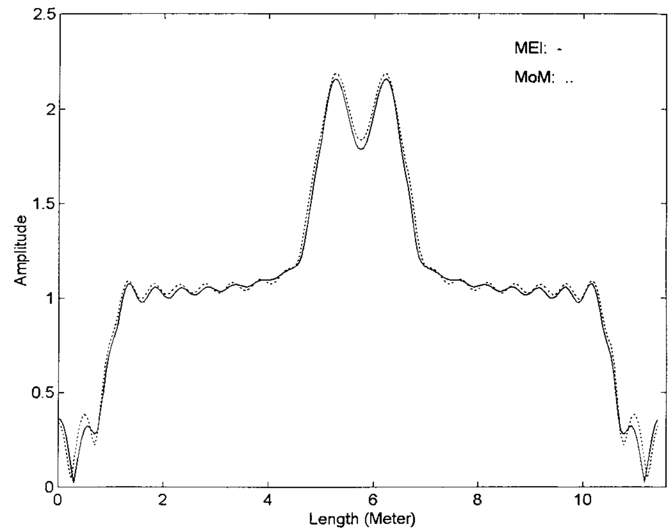

Fig. 5. TE surface currents of the rectangular cylinder.

III. Numerical Results

The first example is the scattering of a conducting circular cylinder with 2λ diameter. In order to ensure calculation accuracy, the ratio of wavelength λ to discretized space step h is kept in $\lambda/h = 20$. A plane wave with the wavelength of 1 m and the incident angle of $0°$ is applied as the incident wave. When node number $N = 3$ in (4), Figs. 2 and 3 show that the total surface currents of the OSMEI agree well to those of the MoM for TM and TE cases, respectively.

The second example is the scattering of a rectangular cylinder with 4λ length and 2λ width under the excitation of an $0°$ incident plane wave with 1-m wavelength. When $N = 3$, Figs. 4 and 5 show that total surface currents of the OSMEI agree well to those of the MoM for TM and TE cases, respectively.

It should be noted that the MEI coefficients used in the TE case are the same coefficients found in the TM case.

IV. Conclusion

The OSMEI method introduced in this letter is a fast computational method for perfectly conducting cylindrical scattering problems. We hope that the interpolation and extrapolation technique for the MEI coefficients in the conventional MEI method [8] can be directly used in the OSMEI method to solve the problem of very electrically large object scatterings.

References

[1] B. Z. Steinberg and Y. Leviatan, "On the use of wavelet expansions in the method of moments," *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 610–619, May 1993.

[2] C. Hafner, *The Generalized Multipole Technique for Computational Electromagnetics*. Boston, MA: Artech House, 1990.

[3] F. X. Canning, "Improved impedance matrix localization method," *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 659–667, May 1993.

[4] K. K. Mei, R. Pous, Z. Q. Chen, Y. W. Liu, and M. Prouty, "The measured equation of invariance: A new concept in field computation," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 202–214, Mar. 1994.

[5] J. M. Rius, R. Pous, and A. Cardama, "Integral formulation of the measured equation of invariance: A novel sparse matrix boundary element method," *IEEE Trans. Magn.*, vol. 32, pp. 962–967, May, 1996.

[6] B. Stupfel and R. Mittra, "A theoretical study of numerical absorbing boundary conditions," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 478–487, May 1995.

[7] N. Morita, N. Kumagai, and J. R. Mauts, *Integral Equation Methods for Electromagnetics.* Boston, MA: Artech House, 1990.

[8] Y. W. Liu, K. N. Yung, and K. K. Mei, "Interpolation, extrapolation and application of the measured equation of invariance to scattering by very large cylinders," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 1325–1331, Sept. 1997.